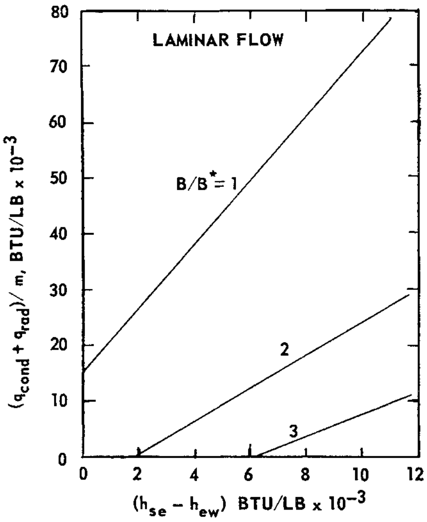

Fig 4 The portion of the effective heat of ablation due to heat conduction and radiation

tion actually produced (usually equal to unity for diffusion-controlled graphite combustion); Γ_{O_e} is the mass fraction of oxygen at the outer edge of the boundary layer; λ_w is the heat of vaporization at surface conditions; q_{cond} is the heat conducted into the graphite; $q_{\ d}$ is the heat radiated from the graphite surface; and η is the blowing effectiveness parameter

$$\eta = (1 - c_{f_e}/c_{f_{e0}})/B(c_f\ /c_{f\ 0}) \tag{6}$$

For graphite ablation, η is 0 68 for laminar flow and 0 39 for turbulent flow From Ref 3, λ_w = 25,300 Btu/lb, Q_{pOw} = 30,200 Btu/lb, and Γ_O = 0 23144

The parameter $Q^*[1 - (q_{\ ad} + q_{cond})/q_0]$ is that portion of Q^* which can be attributed to the ablation process This is presented in Fig 3 for laminar flow as a function of $(h_s - h_{ew})$ with B/B^* as a parameter It can be seen that this quantity is negative for B/B^* less than 1 5 or so With this the case, the net effect of phase change, chemical reaction, and mass addition to the boundary layer is to increase the heat transfer that must be absorbed by the conduction and radiation processes With B/B^* of 2 to 3, the contribution of ablation to Q^* is 10,000 to 20,000 Btu/lb, considerably higher than that of other ablators

The contribution of conduction and radiation to the effective heat of ablation, $(q_{c\ nd} + q_{\ ad})/\dot{m}$, is presented in Fig 4 This is very important for $B/B^* = 1$, especially at high enthalpy levels However, it approaches zero at B/B^* of 2 to 4 dependent upon the enthalpy level, an upper limit in the magnitude of B/B^* for heat transfer to the surface Correspondingly, the maximum possible Q^* for graphite due to the ablation process alone varies from zero for negligible $(h_s - h_{ew})$ to nearly 20,000 Btu/lb at $(h_{\ e} - h_{\ w})$ of 10,000 Btu/lb

In order to determine Q^* from Eq (4) or Fig 2, it is necessary to know the magnitude of the blowing parameter B When diffusion-controlled surface combustion occurs, this presents no problem as $B = B^*$ This is the case for surface temperatures greater than 2000°F or so but below 4000° to 6400°F for pressures of 10^{-4} to 100 atm, respectively [2] When heat-transfer rates are sufficiently high to produce higher surface temperatures, gas-phase combustion occurs, and B exceeds B^* The ratio B/B^* is presented in Ref 2 as a function of surface temperature and pressure

Since B must be computed anyway, there seems to be little advantage to the use of Q^* for the rapid estimation of graphite ablation rates since they can be computed directly from Eq (3).

Calculations have been performed for the diffusion-controlled combustion and sublimation regimes over a wide range of flight conditions The results are presented in graphical form in Ref 2 These should be useful for the rapid estimation of graphite ablation rates and surface temperatures for radiation equilibrium or transient heat conduction (thin-skin approximation) The results are presented in terms of the local static pressure, the stagnation enthalpy, and the heat flux to a cold wall They apply to any position on the body and to turbulent as well as laminar flow conditions

References

[1] Adams, Mac C , "Recent advances in ablation," ARS J **29**, 625–632 (1959)

[2] Bartlett, E P , "Prediction of graphite ablation rates and surface temperatures," Air Force Flight Test Center FTC-TDR-63-40 (to be published)

[3] Denison, M R and Dooley, D A , "Combustion in the laminar boundary layer of chemically active sublimating surfaces," J Aeronaut Sci **25**, 271–272 (1958); also Aeronutronic Systems Inc TR U-110 (September 23, 1957)

[4] Denison, M R , "The turbulent boundary layer on chemically active ablating surfaces," J Aerospace Sci **28**, 471–479 (1961)

[5] Scala, S M , "The ablation of graphite in dissociated air, Part I, Theory," IAS Reprint 62-154 (June 1962)

Velocity Defect Law for a Transpired Turbulent Boundary Layer

H S Mickley,* K A Smith,† and M D Fraser‡
Massachusetts Institute of Technology, Cambridge, Mass

A RECENT paper[1] has proposed that a friction velocity based upon the maximum shear stress should be employed in the formulation of the velocity defect law appropriate to a transpired turbulent boundary layer The theoretical calculations reported here have extended Clauser's[2] concepts to this case but are restricted to $dp/dx = 0$ The results support this view

It is assumed that the stream function is described by

$$\psi = U_1\tilde{\delta}f(\eta) - \int V_0(x)dx \tag{1}$$

where $\tilde{\delta}$ is an arbitrary length scale and η is $y/\tilde{\delta}$ This stream function then allows the momentum equation to be written as

$$-f''f + \frac{V_0}{U_1\tilde{\delta}'}f'' = \frac{\epsilon}{\rho U_1\tilde{\delta}\tilde{\delta}'}f''' \tag{2}$$

where ϵ is the eddy viscosity (assumed independent of y) and the primes indicate differentiation with respect to either η or x, whichever is appropriate Since $\tilde{\delta}$ is as yet arbitrary, let it be defined by

$$\frac{V_0}{U_1\tilde{\delta}'} = \frac{\epsilon}{\rho U_1\tilde{\delta}\tilde{\delta}'} = \frac{1}{\beta} = \text{const} \tag{3}$$

The boundary conditions are

$$f'(0) = U(0)/U_1 \qquad f(0) = 0 \qquad f'(\infty) \rightarrow 1 \tag{4}$$

The first condition reflects the fact that this analysis is valid only over the outer portion of the boundary layer, and the velocity profile for this region extrapolates to a finite value

Received October 24, 1963 The authors are grateful for the financial assistance provided by the National Science Foundation and for the use of the facilities at the Massachusetts Institute of Technology Computation Center, Cambridge, Mass

* Professor, Department of Chemical Engineering
† Assistant Professor, Department of Chemical Engineering
‡ Research Assistant, Department of Chemical Engineering

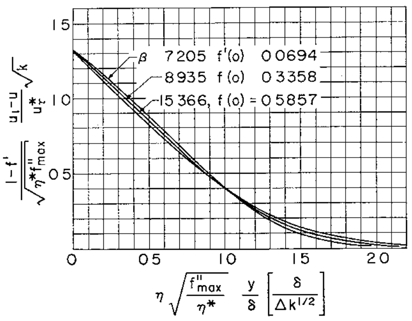

Fig 1 Computed profiles

of U/U_1 at the wall The second is a consequence of Eq (1) The third states that U/U_1 approaches unity asymptotically

Solutions to Eq (2) were obtained on an electronic computer for various combinations of $f'(0)$ (which allows for variation in the distance along the surface x) and β (which allows for variation in the transpiration velocity V_0) Profiles of U/U_1 as a function of η were used to construct defect plots of the form $(U - U_1)/U_\tau{}^*$ vs y/δ, where $U_\tau{}^*$ is a friction velocity based on the *maximum* computed shear stress From Clauser's expression for the eddy viscosity,

$$\epsilon = k\rho U_1 \delta^* \tag{5}$$

it is possible to show that

$$U_\tau{}^* = (\tau^*/\rho)^{1/2} = (kU_1{}^2\eta^* f_{\max}'')^{1/2} \tag{6}$$

where

$$\eta^* = \int_0^\infty \left(1 - \frac{U}{U_1}\right) d\eta$$

Thus,

$$\frac{U_1 - U}{U_\tau{}^*} = \frac{1 - f'}{(k\eta^* f_{\max}'')^{1/2}} \tag{7}$$

The coordinate η may be transformed into y/δ with the aid of a relationship given by Clauser:

$$\frac{y}{\delta} = \frac{\Delta}{\delta}\left(\frac{\Delta\ kf_{\max}''}{\eta^*}\right)^{1/2} \eta \tag{8}$$

where

$$\frac{\Delta}{\delta} = \int_0^1 \frac{U_1 - U}{U_\tau{}^*} d\left(\frac{y}{\delta}\right)$$

The transformed coordinates contain the unknown quantities k and Δ/δ, which may be determined only by experiment However, if an equilibrium layer is, in fact, to exist, these quantities must be constant for a given equilibrium profile and may therefore be deleted for the purpose of determining the existence of such profiles

Figure 1 presents some of the results in terms of these modified coordinates It is apparent that all three of these curves could be satisfactorily approximated by a single universal profile Additional results show this to be true for a very wide range of β and $f'(0)$ Moreover, the effect of β is found to be very weak, the spread between the curves being associated almost entirely with the changes in $f'(0)$ Thus, in terms of the modified, transformed coordinates, nearly all solutions lead to a single defect plot It is therefore not possible, on analytical grounds, to select a similarity parameter analogous to Clauser's $\delta^* dp/\tau_0 dx$ Significantly, the use of other scale velocities, such as a friction velocity based on the wall shear-stress, did not lead to a universal profile

In the authors' judgment, Butensky's data[3] provide the best shear-stress profiles presently available for a transpired turbulent boundary layer Unfortunately, his experimental conditions did not yield a universal profile Nevertheless, if one sets $k = 0\ 018$ as found by Clauser and uses the value of Δ/δ associated with a particular profile, the analytical prediction of that experimental profile is quite satisfactory, as illustrated in Fig 2 The defect law for a nontranspired boundary layer is shown for comparison, and the dashed lines indicate the spread of the theoretical results

It remains for experiment to determine what condition leads to an equilibrium profile, i e , constant Δ/δ This work is now underway in the authors' laboratory

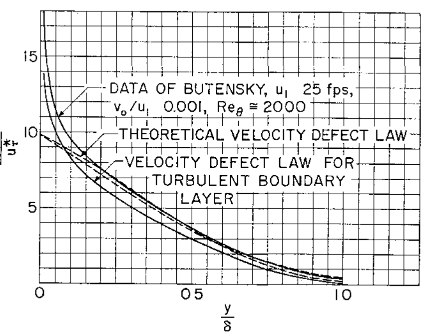

Fig 2 Comparison with experimental data

References

[1] Mickley, H S and Smith, K A , "Velocity defect law for a transpired turbulent boundary layer," AIAA J **1**, 1685–1687 (1963)

[2] Clauser, F H , "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press Inc , New York, 1956), Vol 4, pp 1–51

[3] Butensky, M S , "The transpired turbulent boundary layer on a flat plate," Sc D Thesis, Chem Eng Dept , Mass Inst Tech (1962)

Elastic Stability of Castellated Plates

M Sevik*

Pennsylvania State University, University Park, Pa

IN the design of aircraft wing ribs or fuselage frames, it is often necessary to connect the various structural elements by a plate containing numerous cutouts for stringers and longerons (Fig 1) Such castellated plates are loaded in shear and usually fall because of buckling of the free edge

A data sheet[1] issued by the Royal Aeronautical Society in England enables the calculation of the allowable load on castellation teeth of various geometric proportions The information appearing on this data sheet summarizes the results of a large number of tests The purpose of this note is to show that the shear stress corresponding to initial buckling of the teeth can readily be predicted theoretically and that it is in good agreement with experimental data

We describe the lateral deflection of a castellation by the expression

$$w = A \sin(\pi x/h) \sin(\pi y/2b) \tag{1}$$

The shape represented by Eq (1) is in close agreement with experimental observations The value of w is zero along three sides consisting of the tension edge ($y = 0$) and the root and skin lines ($x = 0$ and $x = h$) It is a maximum at the midpoin t of the compression edge.

Received October 30, 1963

* Assistant Professor of Aeronautical Engineering, Ordnance Research Laboratory